

# Towards a Game-theoretic Notion of Incoercibility

joint work with Dominique Unruh

# Disclaimer

- towards, as in: not there yet.
- started as M.Sc. thesis, extended for Crypto 2011
- found flaw afterwards
- (absolute) definition is incomplete, fix is unsound
- new proposal for relative notion, but how to tell it is correct?

# Motivation: voting schemes

- you know this introduction :)
- for me, personally: what are the things we expect from voting? Can we learn more about pen-and-paper elections?
- won't talk about universal/individual verifiability

# From secrecy to coercion resistance

- **vote-privacy:** protocol does not reveal your vote
- **receipt-freeness:** voter cannot produce receipt
- **coercion-resistance:** voter cannot produce receipt, even if interacting with coercer

# From secrecy to coercion resistance



Andrea Ypsilanti, chairman  
of social-democratic party  
in Hess, 2008

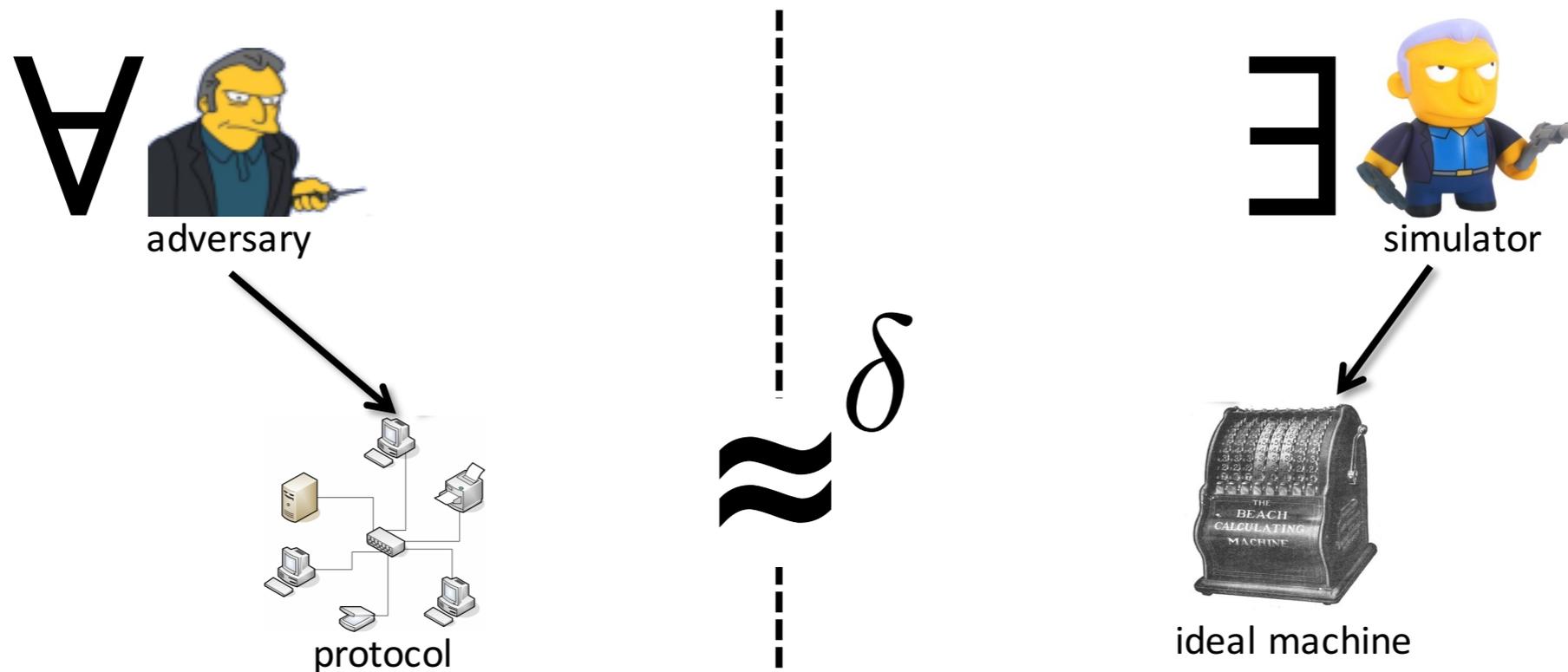
asked/pressured MPs to take  
photograph of ballot.

# Motivation: game theory

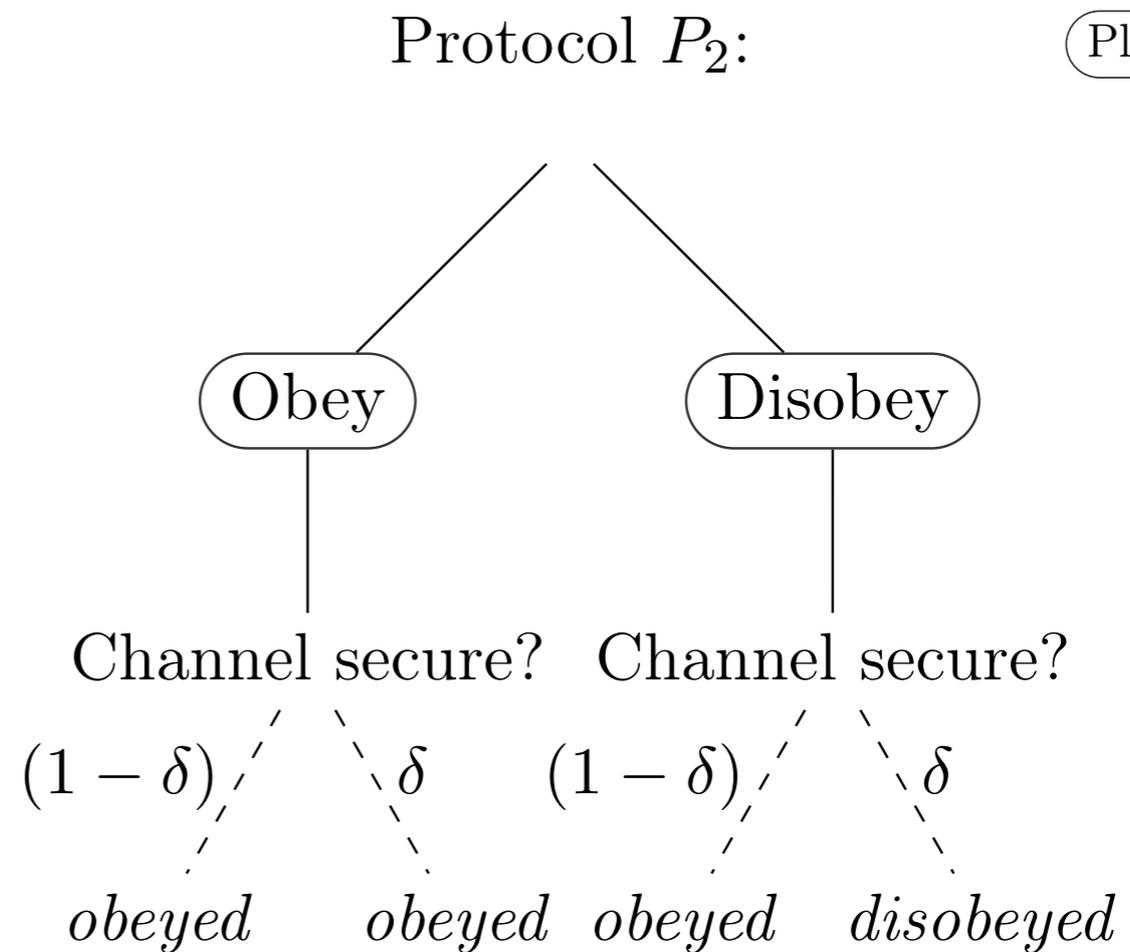
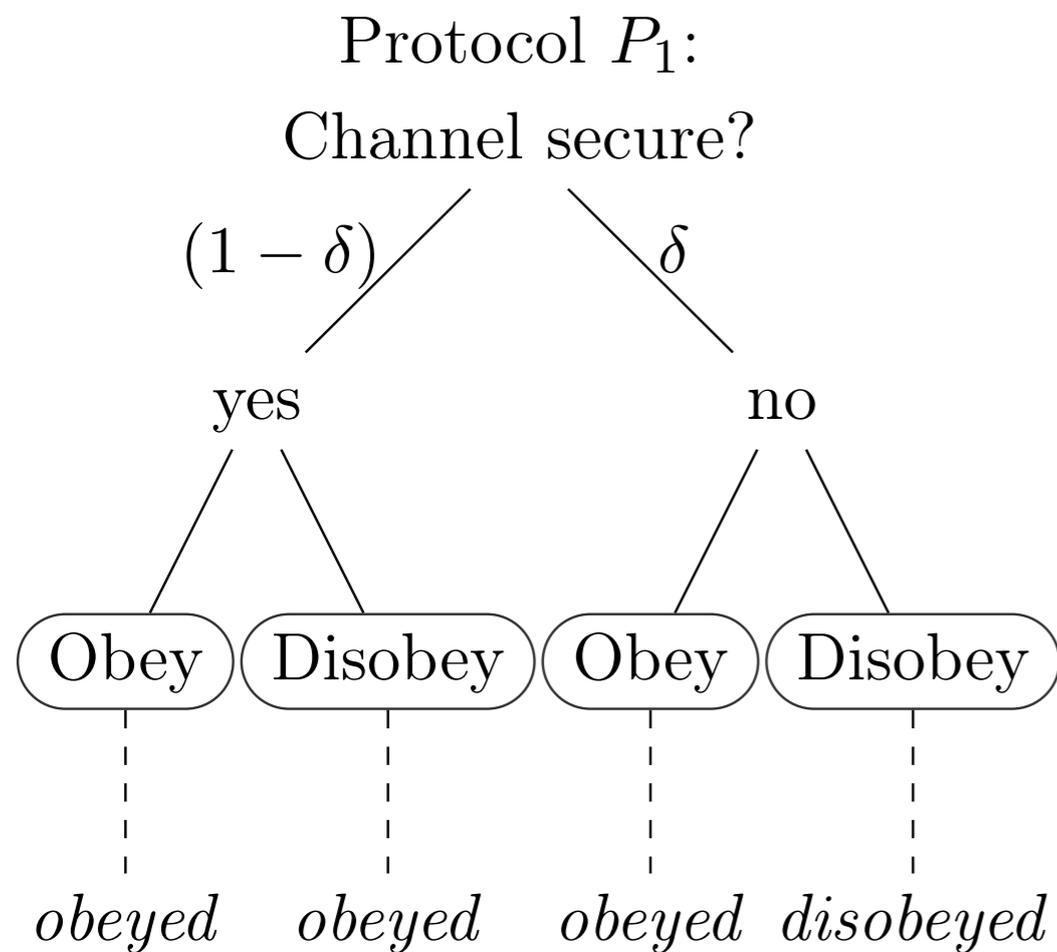
- existing definitions (KTV 2010, UM-Q 2010): there is a *deception strategy* which
  - achieves the good thing (e.g., votes for I)
  - is indistinguishable from a voter which "obeys"

# UC/c

- aesthetically pleasing for people who like UC



# Announced randomness



Player's Decision

----- output

# Game-theoretic framework

- Two player player one-shot game with TMs as strategies:
  - players: P,C
  - output of game: outcome x punishment
  - utility functions for P/C,  $u: \text{outcome} \rightarrow \mathbb{R}$
  - punishment cost:  $p: \text{punishment} \rightarrow \mathbb{R}$

# Equilibria

- most famous: Nash equilibrium
- best response to  $M_C$ :  $M_P$ , s.t.  $u(M_C, M_P) \geq u(M_C, M_{P'})$
- everyone plays the best response to everyone else
  
- approach: coercion is in equilibrium
- best response to an effective coercer is obeying
- best response to an obeying player is: not coercing....

# Best response for player

- deception strategy may depend on coercer:
  - best-response is a nice measure
- but goal should still be reached
- define disarmed/threat-utility as expected value of
  - $u^{\text{dis}}$ : based on outcome
  - $u^{\text{thr}}$ :  $u^{\text{dis}}$  - punishment

# Harmless coercer

- a coercer is harmless iff best-response with and without threat have same disarmed utility
- Example:  $u(\text{Clinton})=5$   $u(\text{Trump})=0$   $u(\text{Stein})=10$
- protocol incoercible if for all (reasonable) utility functions, all coercers are harmless.

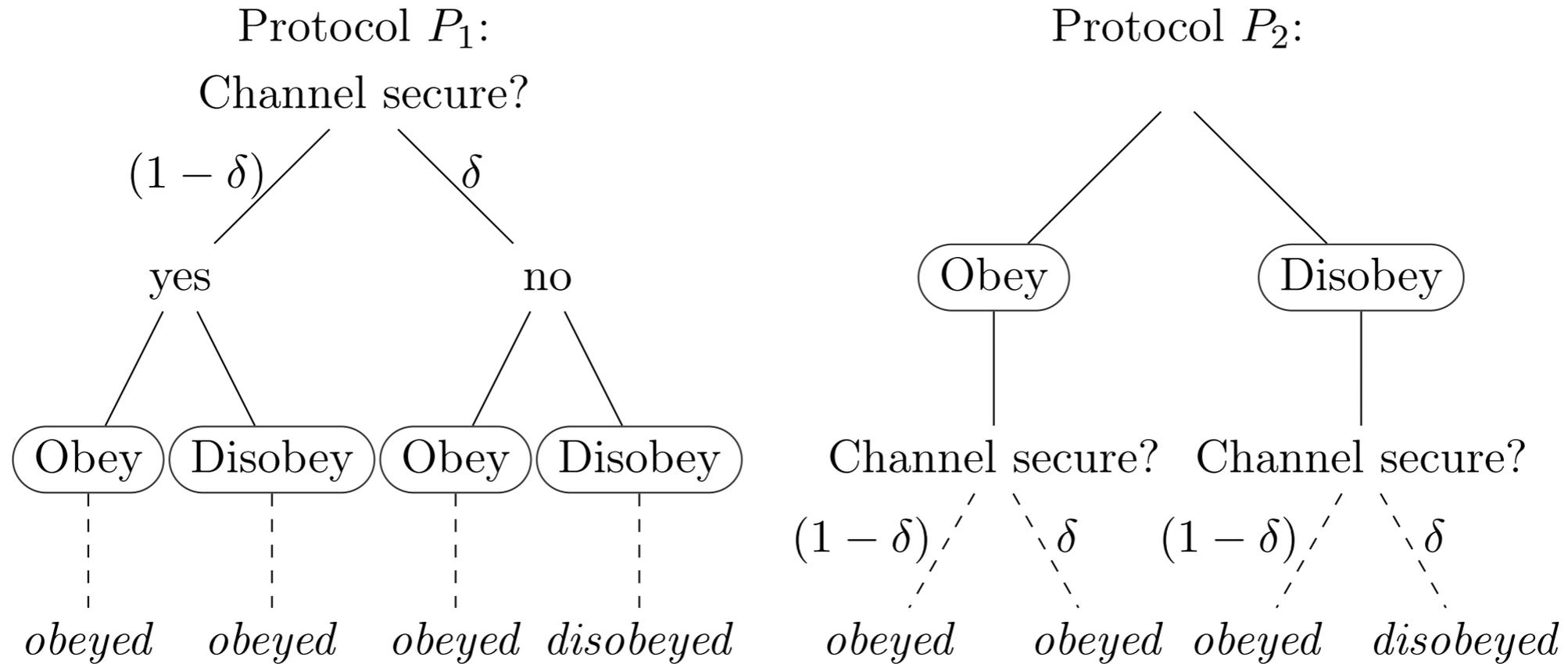
# Allen Wertheimer

- A. *The Proposal Prong.* A threatens B by **wrongfully** (viz. without moral justification) making B a **proposal** such that, unless B complies, B will be in a **worse position** than B was **otherwise entitled to expect to be**; and
- B. *The Choice Prong.* B is morally justified in complying (e. g., since **there is no reasonable alternative**) and does **comply** with A's proposal.

$$\text{for all } M_P \in BR^{\text{thr}}(M_C) \quad M'_P \in BR^{\text{dis}}(M_C) \\ U_P^{\text{dis}}(M'_P, M_C) - U_P^{\text{dis}}(M_P, M_C) < \delta$$

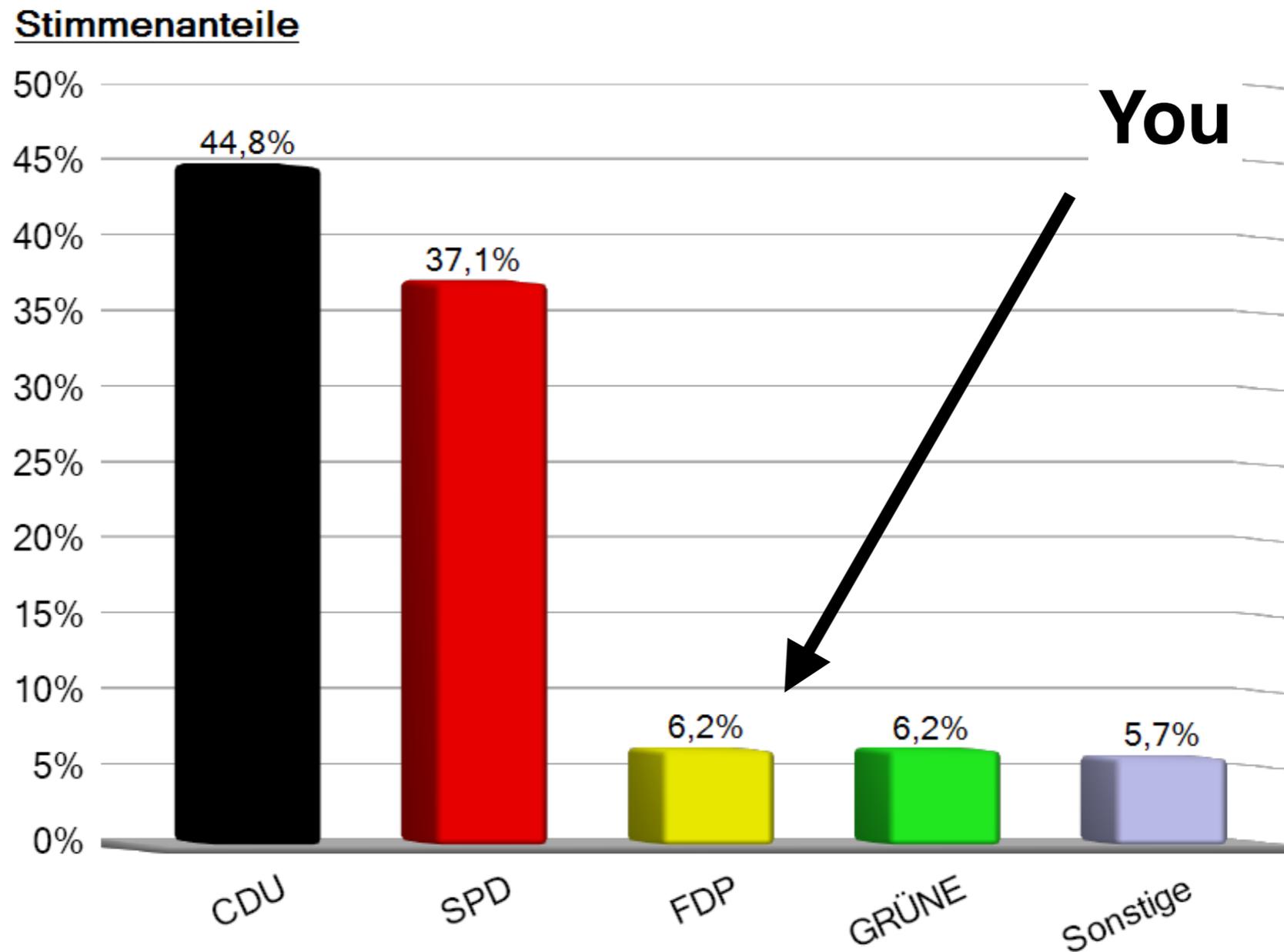
# Results

- definition grounded in Wertheimer's Definition
- relative variant:  $P$   $\delta$ -as-good-as  $Q$  if for same utility function, same loss of disarmed utility (up to  $\delta$ )
- connection to UC/c: if  $P$  emulates ideal voting functionality, then it is 0-as-good-as it.
- connection to KTV: if  $P$  is  $\delta$ -KTV-incoercible, and  $u_{\text{dis}}$  smaller  $w$ , then  $\min\{w, \delta\}$ -incoercible (absolute).
- counter-example KTV: coercer makes his vote dependent via secret
- analysis of ideal voting in election district Saarbrücken: value of objective needs to be higher than 4% of punishment.



	$G_1$	$G_2$	Thm 10
$1 < w$	0	0	$\delta$
$\delta < w \leq 1$	$\delta \cdot w$	0	$\delta$
$w \leq \delta \leq 1$	$\delta \cdot w$	$w$	$w$

# Analysis Saarbrücken



# Problem: legal coercion

- coercer can vote, too.
- punishment by voting (incompleteness).
  - C votes Stein and punishes if P sends "Hello", otherwise Trump
  - threat-best-response sends Hello, disarmed-best-response not.
  - disarmed best-response has worse pay-off

# Problem: legal coercion

$$\begin{aligned} & \min_{M_P, M_C \text{ with } M_P \in BR^{dis}} U_P^{dis}(M_C, M_P) \\ & \leq \min_{M_P, M_C \text{ with } M_P \in BR^{thr}} U_P^{dis}(M_C, M_P). \end{aligned}$$

- legal coercion masks actual coercion: if voting for Trump is bad enough, receipt-giving protocol counts as incoercible (unsoundness..?).
- coercer can vote Trump to force P into providing receipt
- counter-example for indist.-based notions does not fly anymore

# Proposal f. relative notion

- P is as-good-as Q, if
  - for all  $M_C$  and  $M_P \in BR^{thr}(M_C)$  in P
  - there exist  $M_C'$  and  $M_P' \in BR^{thr}(M_C')$  in Q
  - s.t. the outcome is (often enough) the same
- reflexive, transitive, but if Q is disarmed version of P, legal coercion still masks illegal coercion

# Conclusion

- ?
- game-theoretic foundation for simulation-based notions seem achievable, framework has proven very useful.
- quantifying incoercibility in actual districts is interesting — large variations in size of voting districts in Germany could lead to problems
- approach: rational defender, irrational attacker seems ok (to me) can add economics, budget etc., too.
- maybe masking is ok? look only at utility functions where actual punishment is more severe than punishment by voting behaviour

Thank you for your  
attention.

Ask away!